Understanding Causation and Correlation: Exploring the Relationship with Examples

Authored by

Mohammed looti

November 1, 2025

RECOMMENDED CITATION

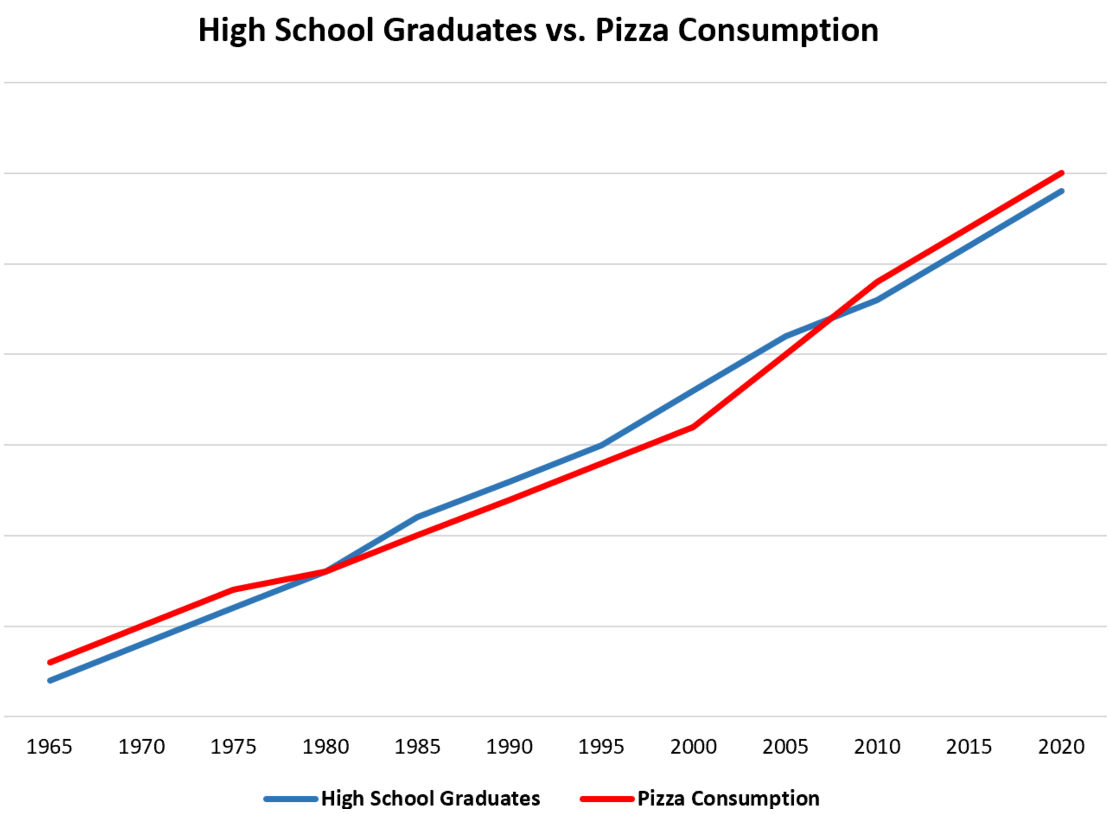
Mohammed looti (2025). *Understanding Causation and Correlation: Exploring the Relationship with Examples*. PSYCHOLOGICAL STATISTICS. Retrieved from https://statistics.arabpsychology.com/?p=8000

In the expansive fields of statistics and data science, one aphorism is repeated as a core safeguard against statistical errors: "**Correlation does not imply causation**." This foundational principle serves as a constant reminder that observing two variables moving in tandem does not automatically prove that one exerts a direct influence upon the other. While this statistical safeguard is broadly understood and taught early in quantitative studies, the inverse relationship poses a far more nuanced and challenging question that often eludes researchers: Does **causation** imply **correlation**?

To properly address this advanced inquiry, it is first necessary to quickly review the risks associated with confusing simple association with direct influence, often illustrated through the concept of a spurious correlation. These are situations where a strong statistical link appears to exist purely by coincidence or, more commonly, due to the unseen force of a third, unmeasured factor, frequently termed a **confounder** or lurking variable.

Revisiting the Classic Fallacy: The Confounding Variable

As a classic, easily digestible illustration of the correlation-causation fallacy, consider annual data collected over several decades in the United States, tracking two seemingly unrelated metrics: the total number of high school graduates and the national volume of pizza consumption. If we analyze this dataset, we would almost certainly find a remarkably high positive statistical correlation.



This strong statistical association means that as the number of graduates increased year after year, so too did the consumption of pizza. However, it would defy both logic and common sense to assert that receiving a high school diploma somehow *causes* people to eat more pizza, or vice versa. There is no plausible biological, economic, or sociological mechanism linking educational attainment directly to increased pizza sales volume in a causal chain.

The true, much more plausible mechanism involves a classic confounding variable: the steady growth of the **U.S. population** over the measured period. As the population expands, two things happen simultaneously: the pool of individuals earning a high school degree increases, and the total volume of all consumed goods, including pizza, also rises. The observed correlation is thus merely a byproduct of this underlying demographic trend, not evidence of a direct causal link between the two primary variables.

The Core Question: Does Causation Imply Correlation?

Having established the limits of correlation as a reliable indicator of causation, we pivot to the more intricate question that forms the core of this discussion: **Does causation imply correlation?** If we possess definitive, deterministic evidence that variable X directly causes a predictable change in variable Y, should we expect to always find a corresponding statistical relationship between them?

The immediate, intuitive answer for many analysts is usually yes. If X perfectly dictates Y--if the relationship is a known functional dependency--it seems counterintuitive that standard statistical measures would fail to detect this powerful connection. Yet, this is precisely where the precise mathematical definition of the term "correlation" becomes critically important and often leads to confusion.

The statistically rigorous answer is a definitive and surprising **No**. The standard methods used to measure linear association can completely miss a perfect causal relationship. The following examples demonstrate that even when **causation** is certain and the relationship is deterministic, the commonly used measure of linear correlation can be exactly zero. This occurs because the standard statistical tools are inherently limited to detecting only straight-line trends.

Understanding the Limits of Linear Correlation

When analysts and statisticians refer to "correlation" without further qualification, they are nearly always referring to the **Pearson correlation coefficient** (r). This coefficient is mathematically designed to quantify the strength and direction of a specific type of relationship: a linear relationship between two continuous variables. The Pearson coefficient measures how closely data points align along a single straight line.

A calculated coefficient close to +1 signifies a strong positive linear association (as X increases, Y increases proportionally in a straight line), while a value near -1 indicates a strong negative linear association. Crucially, a coefficient of 0 does not mean "no relationship exists"; it means there is **no linear relationship** between the variables.

This limitation--its blindness to non-linear associations--is the key insight. If the functional relationship between X and Y follows a curve, a cyclical pattern, or any symmetrical distribution around the mean, the positive deviations from the mean and the negative deviations from the mean will perfectly cancel each other out during the calculation of the Pearson coefficient. This cancellation effect mathematically forces the correlation score to zero, despite the existence of a perfect, undeniable functional dependency--and therefore, a perfect causal link.

Example 1: The Quadratic Relationship ($Y = X^2$)

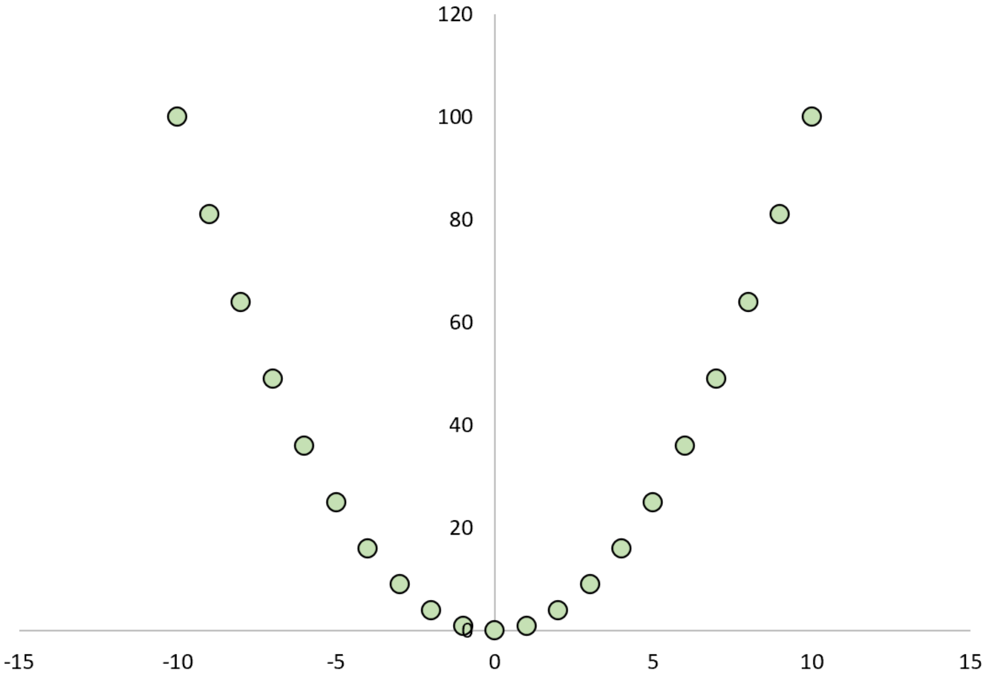
Let us examine a simple scenario where variable X is the sole and deterministic cause of variable Y, defined by the **quadratic** equation: Y = X2. In this model, X perfectly dictates Y; there is no measurement error, no noise, and no external influence. The causal link is explicit and total. If we know X, we know Y.

Consider the influence of X on Y across a symmetrical range of input values. We select values for X that span both the negative and positive sides of the axis to ensure symmetry around the mean:

If X = -10 then Y = (-10)2 = 100
If X = -5 then Y = (-5)2 = 25
If X = 0 then Y = (0)2 = 0
If X = 5 then Y = (5)2 = 25
If X = 10 then Y = (10)2 = 100

As X moves from negative values toward zero, the value of Y decreases, exhibiting a negative slope. Conversely, as X moves from zero toward positive values, the value of Y increases, showing a positive slope. Because these two segments of change perfectly mirror each other across the y-axis, the statistical mechanism used to calculate the linear correlation registers zero net linear movement.

If we plotted this deterministic relationship between X and Y, the resulting parabola clearly shows a strong functional dependency:



Despite the strong functional dependency, visually represented by a perfect parabola on a scatter plot, calculating the linear correlation coefficient (r) for this symmetrical dataset yields an exact value of **zero**. This outcome unequivocally demonstrates that a perfect causal relationship does not necessitate a detectable linear correlation.

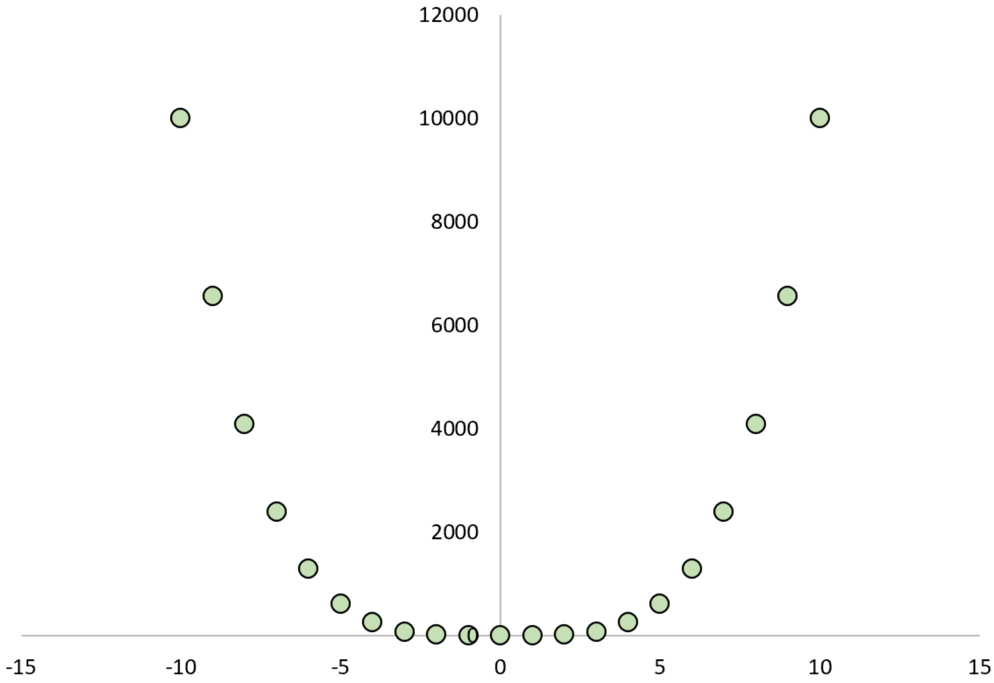
Example 2: The Symmetric Quartic Function (Y = X?)

This principle extends beyond the quadratic function to other even-powered polynomial relationships. Let us consider a causal link defined by the **quartics** function: Y = X4. Similar to the previous example, this function establishes a clear, deterministic causal link: X is the input that precisely dictates the output Y. The relationship is perfectly defined, but distinctly non-linear.

Observing a symmetrical range of points further reinforces the cancellation effect, often with even more extreme changes in slope near the center:

If X = -10 then Y = (-10)4 = 10,000
If X = -5 then Y = (-5)4 = 625
If X = 0 then Y = (0)4 = 0
If X = 5 then Y = (5)4 = 625
If X = 10 then Y = (10)4 = 10,000

The behavior is entirely symmetrical around X = 0. As X moves away from the origin, regardless of direction (positive or negative), Y rapidly increases into the positive range. This extreme symmetry ensures that the positive and negative components of the covariance term within the Pearson correlation coefficient calculation perfectly neutralize one another.



Despite the plot clearly confirming a strong functional dependency--a perfect causal relationship--calculating the linear correlation coefficient (r) between X and Y based on data points symmetrical around zero results in **zero**. This robust mathematical finding confirms that relying solely on a linear measure will fail to detect highly effective, yet non-linear, causal mechanisms.

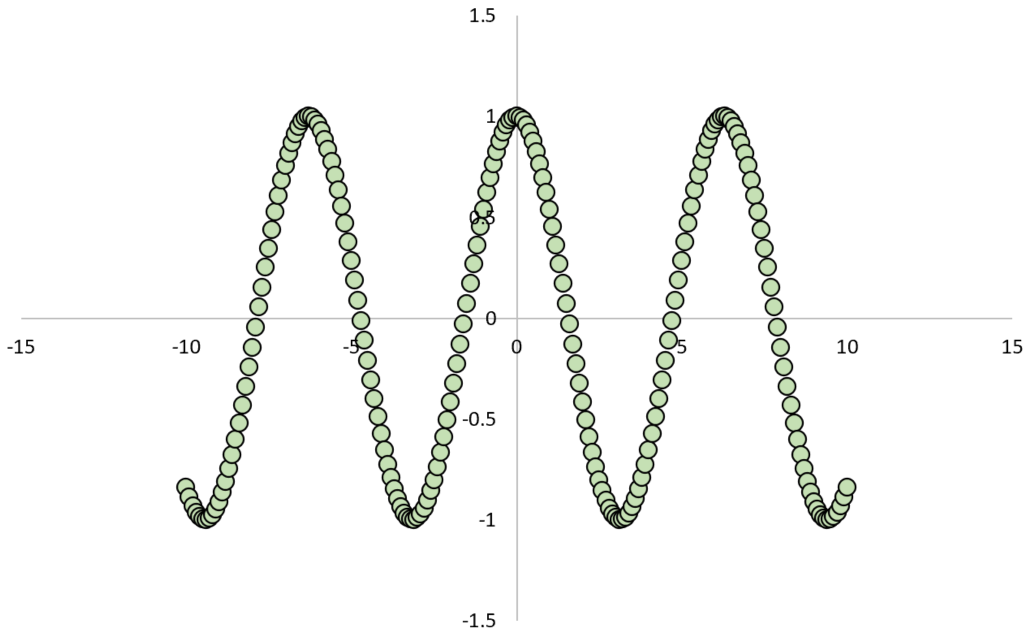
Example 3: The Cyclical Cosine Relationship (Y = cos(X))

To illustrate the problem outside of polynomial equations, we turn to a cyclical, periodic relationship defined by the trigonometric function: Y = cos(X). Here, X still perfectly causes Y, but the output oscillates consistently between -1 and 1 as X increases. This relationship is entirely deterministic, yet fundamentally non-linear.

The deterministic nature of this relationship can be observed across key points:

If X = -10 then Y = cos(-10) ≈ -0.83907
If X = -2π then Y = cos(-2π) = 1
If X = 0 then Y = cos(0) = 1
If X = 2π then Y = cos(2π) = 1
If X = 10 then Y = cos(10) ≈ -0.83907

When analyzing data spanning multiple cycles of the cosine wave, the relationship constantly shifts its slope. There are periods where X increases and Y decreases (a negative slope), immediately followed by periods where X increases and Y also increases (a positive slope). This constant, offsetting oscillation and periodicity ensures that, over a wide range of X values, the overall linear relationship is neutralized.



As with the polynomial examples, if the Pearson correlation coefficient is calculated for X and Y across a sufficiently wide range of inputs, the resulting correlation will be **zero**. This provides compelling evidence that while X perfectly dictates Y (perfect **causation**), the required straight-line association is entirely absent.

Implications for Statistical Modeling and Analysis

The crucial finding that **causation** does not necessarily imply linear correlation holds profound implications for how researchers and data analysts approach exploratory data analysis and modeling. If an analyst relies solely on calculating the Pearson coefficient to screen for potential relationships, they run the significant risk of discarding variables that are, in fact, perfectly related.

For instance, if an analyst finds a Pearson correlation value near zero, they might erroneously conclude that there is absolutely no relationship between the two variables. Yet, as demonstrated by the quadratic, quartic, and cosine functions, a strong, deterministic, and highly predictive non-linear relationship could be present, operating entirely undetected by the linear measure.

This limitation underscores the critical necessity of adopting robust exploratory techniques beyond simple correlation tables. Analysts must always **visually inspect the data** first. Creating scatter plots immediately reveals the true shape of the relationship, allowing the analyst to identify non-linear patterns (curves, cycles, or symmetrical distributions) that the linear correlation measure actively obscures. Finding a zero correlation only indicates the absence of a straight-line trend; it provides no information whatsoever about complex causal curves or cyclical dependencies.

Additional Resources

The following resources provide additional information about the distinction between correlation and causation: