

# Understanding Bernoulli Trials: Success and Failure Conditions in Statistics

Authored by  
**Mohammed loot**

November 7, 2025

## RECOMMENDED CITATION

Mohammed loot (2025). *Understanding Bernoulli Trials: Success and Failure Conditions in Statistics*. PSYCHOLOGICAL STATISTICS. Retrieved from <https://statistics.arabpsychology.com/?p=12225>

In the realm of statistics, especially when analyzing categorical data, the concept of a trial with only two possible outcomes is fundamental. This elementary experiment is known as a [Bernoulli trial](#). By definition, a Bernoulli trial is characterized by having exactly two mutually exclusive results--conventionally labeled as "success" or "failure"--and maintaining a constant probability of success ( $p$ ) across every repetition. Understanding this foundational concept is crucial, as it leads directly to more complex distributions used in approximation techniques.

A classic example used to illustrate this principle is the simple coin flip. If we designate "Heads" as a **success** and "Tails" as a **failure**, the probability of success remains 0.5 for each independent flip (assuming a fair coin). When we aggregate a fixed number of these independent Bernoulli trials, we are then dealing with the [binomial distribution](#). This distribution allows us to calculate the exact probability of observing a specific number of successes ( $x$ ) in a given number of trials ( $n$ ). However, calculating these exact probabilities can become extremely tedious and computationally expensive when the number of trials,  $n$ , is large.

## Foundations: The Bernoulli Trial and the Binomial Context

The binomial distribution is defined by two primary parameters: the sample size,  $n$  (the number of trials), and the probability of success,  $p$ . While the binomial model is precise for modeling discrete outcomes, its computational complexity historically necessitated the use of approximations, particularly before advanced computing was readily available. Even today, using an approximation simplifies many theoretical and practical calculations, especially those involving inferential statistics like hypothesis testing or the construction of confidence intervals for population proportions.

As the number of trials  $n$  increases, the discrete shape of the binomial distribution gradually begins to resemble the continuous, bell-shaped curve of the [normal distribution](#). This convergence is a powerful phenomenon that allows statisticians to use the familiar and well-tabulated properties of the normal curve--such as Z-scores--to estimate binomial probabilities. However, this approximation is only mathematically valid when the binomial distribution is sufficiently symmetrical and spread out. This critical requirement brings us to the necessity of validating the **Success/Failure Condition**.

If the probability of success,  $p$ , is extremely close to 0 or 1, the binomial distribution will be highly skewed. For instance, if  $p = 0.01$ , most outcomes will be failures, clustering the distribution near zero successes, which is far from the symmetrical shape of the normal curve. The **Success/Failure Condition** serves as a robust check to ensure the distribution is centered enough, with adequate spread on both sides, to justify the use of the normal approximation without introducing excessive error.

## Defining the Success/Failure Condition for Normal Approximation

The core purpose of the **Success/Failure Condition** is to ensure that the sample size ( $n$ ) is large enough relative to the probability of success ( $p$ ) to guarantee that the resulting distribution of successes is reasonably close to the normal model. If this condition is violated, any statistical inference based on the normal distribution (such as a Z-test or a Z-interval) will be unreliable and potentially misleading.

The condition itself is straightforward, focusing on the expected counts rather than the actual observed counts in the sample. It requires that we have a minimum predicted frequency for both outcomes (success and failure). This dual requirement prevents the distribution from being heavily skewed in either direction, ensuring a more bell-like structure that is suitable for normal approximation.

The formal definition, which acts as a gatekeeper for statistical methods relying on the normal model for proportions, is stated as follows:

**Success/Failure Condition:** There should be at least 10 expected successes and 10 expected failures in a sample in order to use the normal distribution as an approximation for the binomial distribution when performing inference on proportions.

## Mathematical Formulation and Interpretation

To verify the **Success/Failure Condition** mathematically, we translate the expected counts into simple inequalities involving the sample size ( $n$ ) and the probability of success ( $p$ ). The expected number of successes is calculated as the product of the number of trials and the probability of success ( $np$ ). Correspondingly, the expected number of failures is calculated by multiplying the number of trials by the probability of failure ( $1-p$ ), resulting in  $n(1-p)$ .

For the condition to be satisfied, both expected counts must meet or exceed the specified threshold of 10. This requirement must be verified prior to calculating any statistics that depend on the normal model, such as the standard error used in confidence interval calculations. If even one of these inequalities is not met, the underlying data distribution is considered too skewed, and alternative methods (like exact binomial tests) should be employed instead of the normal approximation.

We must therefore verify both of the following criteria:

Expected number of successes is at least 10:  $np \geq 10$

Expected number of failures is at least 10:  $n(1-p) \geq 10$

The choice of the number 10 is a conservative statistical standard. While the convergence of the

binomial to the normal distribution happens theoretically as  $n$  approaches infinity, the threshold of 10 for both expected counts is a pragmatic rule of thumb. It ensures that the tails of the discrete distribution do not "run out" prematurely, meaning there are enough data points on both sides of the mean to adequately capture the shape required for continuity correction and normal model accuracy. Using a smaller threshold, such as 5, may lead to acceptable results in some scenarios but generally increases the risk of calculation error, especially when  $p$  is far from 0.5.

## Practical Application: Checking the Condition for a Confidence Interval

Consider a scenario where a political analyst wishes to estimate the true proportion of residents in a large county who support a new environmental law. They select a random sample of 100 residents ( $n = 100$ ) and find that 56% are in favor of the law. To construct a [confidence interval](#) for the population proportion, the analyst intends to use the standard Z-interval formula, which inherently relies on the normal distribution as an approximation of the underlying binomial data.

The initial sample statistics are:

Sample size  $n = 100$

Sample proportion in favor of the law  $p = 0.56$

The formula for the confidence interval for a proportion is typically written as:

$$\text{Confidence Interval} = p \pm z \sqrt{p(1-p)} / n$$

Because this formula utilizes the Z-value (the critical value derived directly from the standard normal distribution), we must first verify that the **Success/Failure Condition** is met. We calculate the expected number of successes and failures based on our sample size and proportion:

$$\text{Number of successes: } np = 100 * 0.56 = 56$$

$$\text{Number of failures: } n(1-p) = 100 * (1 - 0.56) = 100 * 0.44 = 44$$

Since the expected number of successes (56) is greater than or equal to 10, and the expected number of failures (44) is also greater than or equal to 10, the **Success/Failure Condition** is satisfied. This confirmation allows the analyst to confidently proceed with the calculation of the confidence interval using the normal approximation formula, knowing the approximation error will be minimal.

## Contextual Considerations and Alternative Rules

It is important for statistical practitioners to recognize that the 10/10 rule ( $np \geq 10$  and  $n(1-p) \geq 10$ ) is not universally mandated across all textbooks or statistical contexts. Some introductory

resources, prioritizing ease of calculation, suggest a more lenient rule, often requiring only 5 expected successes and 5 expected failures. However, the 10/10 standard is widely adopted in practice, particularly in professional statistical analysis, due to its increased rigor and reliability in ensuring the distribution is truly symmetrical enough for the normal model to be applicable.

Beyond the inherent requirements for approximation, two other conditions frequently accompany the Success/Failure Condition when conducting inference on proportions:

**Randomization Condition:** The data must come from a randomized, well-designed sample or experiment. Without proper randomization, the data may be biased, rendering the subsequent statistical calculations irrelevant.

**The 10% Condition:** When sampling is done without replacement (which is typical), the sample size  $n$  must not exceed 10% of the total population size  $N$ . This ensures that the trials remain sufficiently independent, allowing us to treat the probabilities as constant throughout the sampling process. If this condition is violated, the calculated standard error will be too large, requiring the use of a finite population correction factor. Failure to meet the [10% Condition](#) significantly compromises the independence assumption that underlies the binomial model.

Furthermore, when performing statistical inference involving the comparison of two different population proportions (e.g., comparing support for a law in two different districts), the Success/Failure Condition must be met for **both** samples independently. This means four separate inequalities must be checked: the expected successes and failures for Sample 1, and the expected successes and failures for Sample 2. Meeting these conditions ensures that the sampling distributions derived from both sets of data are suitable for using the pooled normal model required for two-sample proportion tests or confidence intervals for the difference in proportions.